**Faculty of Computer Science, Dalhousie University** *06-Nov-2025*

# CSCI 4152/6509 — Natural Language Processing

# Lecture 13: P0 Project Topics Discussion (1); HMM Model

Location: Studley LSC-Psychology P5260 Instructor: Vlado Keselj
Time: 14:35 – 15:55

**Previous Lecture**

- N-gram Model Smoothing (continued):
  - Witten-Bell smoothing
- **POS tagging: Introduction**
- Open word categories
- Closed word categories
- Other word categories
- **Hidden Markov Model (HMM):**
  - idea, definition, graphical representation
  - HMM assumption
- HMM POS Example

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**P0 Topics Discussion**

- Discussion of individual projects as proposed in P0 submissions
- Projects discussed: P-03, P-07, P-08, P-09, P-11, P-12, P-20, P-21, P-22, P-23

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*Slide notes:*

**Learning HMM (Training) with Smoothing**

- Let us learn HMM from completely labeled data:

```
swat V flies N like P ants N
time N flies V like P an D arrow N
```

- We will use smoothing in word generation, by giving a 0.5 count to all unseen words

Having this in mind, suppose that we are given the following training data:

```
swat V flies N like P ants N
time N flies V like P an D arrow N
```

To accommodate for unseen words, we can assign a special symbol $\star$ to unknown words, and assume that it occurred 0.5 "times" with each tag.

First, we can "learn" the probability of initial states $\pi$ $(= \mathrm{P}(T_1)$ by counting how many times each state was the first state in a sequence, and obtain the following counts and the resulting probabilities:

| $q$ | counts for $\pi(q)$ |
|---|---|
| D | 0 |
| N | 1 |
| P | 0 |
| V | 1 |

$\Rightarrow$

| $q$ | $\pi(q)$ |
|---|---|
| D | 0 |
| N | 0.5 |
| P | 0 |
| V | 0.5 |

We will also denote the initial probability $\pi(q)$ as $\pi(q) = \mathrm{P}(T_1 = q)$.

Similarly, we count the transitions in order to estimate transition probabilities $a$:

| counts for $a(p,q)$ | D | N | P | V | sum |
|---|---|---|---|---|---|
| D | 0 | 1 | 0 | 0 | 1 |
| N | 0 | 0 | 1 | 1 | 2 |
| P | 1 | 1 | 0 | 0 | 2 |
| V | 0 | 1 | 1 | 0 | 2 |

$\Rightarrow$

| $a(p,q)$ | D | N | P | V |
|---|---|---|---|---|
| D | 0 | 1 | 0 | 0 |
| N | 0 | 0 | 0.5 | 0.5 |
| P | 0.5 | 0.5 | 0 | 0 |
| V | 0 | 0.5 | 0.5 | 0 |

We will also denote the transitional probability $a(p,q)$ as $a(p,q) = PP(T_{i+1} = q | T_i = p)$, which more clearly presents meaning of this probability table.

We will incorporate our smoothing method into learning of the output probabilities by giving a count of 0.5 to any unseen words, marked with '*', to be generated from any tag. This is how we obtain the following counts:

| counts for $b(q,o)$ | an | ants | arrow | flies | like | swat | time | * | sum |
|---|---|---|---|---|---|---|---|---|---|
| D | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 1.5 |
| N | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0.5 | 4.5 |
| P | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0.5 | 2.5 |
| V | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0.5 | 2.5 |

which leads to the following estimated probabilities obtained by dividing numbers in each row with the corresponding sum:

| $b(q,o)$ | an | ants | arrow | flies | like | swat | time | * |
|---|---|---|---|---|---|---|---|---|
| D | $2/3$ | 0 | 0 | 0 | 0 | 0 | 0 | $1/3$ |
| N | 0 | $2/9$ | $2/9$ | $2/9$ | 0 | 0 | $2/9$ | $1/9$ |
| P | 0 | 0 | 0 | 0 | $4/5$ | 0 | 0 | $1/5$ |
| V | 0 | 0 | 0 | $2/5$ | 0 | $2/5$ | 0 | $1/5$ |

Another way to represent the learned table is as the following conditional probability tables (CPTs):

**Generated Tables**

| $T_1$ | $\mathrm{P}(T_1)$ |
|---|---|
| N | 0.5 |
| V | 0.5 |

,

| $T_{i-1}$ | $T_i$ | $\mathrm{P}(T_i \| T_{i-1})$ |
|---|---|---|
| D | N | 1 |
| N | P | 0.5 |
| N | V | 0.5 |
| P | D | 0.5 |
| P | N | 0.5 |
| V | N | 0.5 |
| V | P | 0.5 |

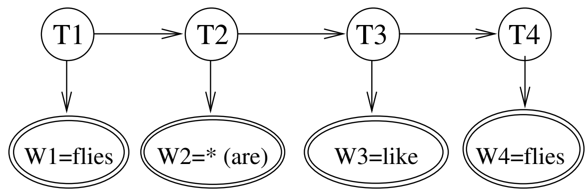
and

| $T_i$ | $W_i$ | $\mathrm{P}(W_i \| T_i)$ |
|---|---|---|
| D | an | $2/3 \approx 0.666666667$ |
| D | * | $1/3 \approx 0.333333333$ |
| N | ants | $2/9 \approx 0.222222222$ |
| N | arrow | $2/9 \approx 0.222222222$ |
| N | flies | $2/9 \approx 0.222222222$ |
| N | time | $2/9 \approx 0.222222222$ |
| N | * | $1/9 \approx 0.111111111$ |
| P | like | 0.8 |
| P | * | 0.2 |
| V | flies | 0.4 |
| V | swat | 0.4 |
| V | * | 0.2 |

.

In the tables above we did not include zero-probabilities: for example, $\mathrm{P}(T_i = V|T_{i-1} = D)$ is not included since it is equal to 0. The symbol '*' is used to denote any unseen word that may appear in a testing sentence.

Let us use the Hidden Markov Model to POS tag the sentence "flies are like flies."




The problem of POS tagging in this case is the problem of finding the most probable values of the variables $T_i$ given the values of variables $W_i$, i.e., it is the completion problem of finding

$$\begin{aligned}\arg\max_T \mathrm{P}(T|W = \text{sentence}) &= \arg\max_T \frac{\mathrm{P}(T, W = \text{sentence})}{\mathrm{P}(W = \text{sentence})} = \arg\max_T \mathrm{P}(T, W = \text{sentence}) \\ &= \arg\max_T \mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1) \cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = *|T_2) \\ &\quad \cdot \mathrm{P}(T_3|T_2) \cdot \mathrm{P}(W_3 = \text{like}|T_3) \cdot \mathrm{P}(T_4|T_3) \cdot \mathrm{P}(W_4 = \text{flies}|T_4)\end{aligned}$$

where $T$ and $W$ denote arrays of variables $T_i$ and $W_i$. One way to find the values of $T$ that maximize the given probability is to test all variations of their values. This number is exponential in general, and in this case it is $4^4 = 256$. A much more efficient solution can be obtained by applying a dynamic programming approach, known as the Viterbi algorithm.

**"Brute-Force" Approach**

- Try all combinations of variable values $T_1$, $T_2$, $T_3$, and $T_4$
- Calculate the overall probability for each of them using the formula

$$\begin{aligned}&\mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1) \\ &\cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = *|T_2) \\ &\cdot \mathrm{P}(T_3|T_2) \cdot \mathrm{P}(W_3 = \text{like}|T_3) \\ &\cdot \mathrm{P}(T_4|T_3) \cdot \mathrm{P}(W_4 = \text{flies}|T_4)\end{aligned}$$

- Choose the maximal probability

## 15.3 Efficient Tagging with HMM

**Efficient Tagging with HMM**

- Rather than using the brute-force approach, we can incrementally optimize the product expression by partial maximization from left to right
- One way to represent this is by using a table, which leads to the dynamic programming solution, or the Viterbi algorithm
- The second way to represent this computation is using message passing, or product-sum algorithm

**HMM Inference: Dynamic Programming Solution**

- Brute-force approach is too inefficient
- Idea for more efficient calculation: maximize sub-products first

- Dynamic Programming approach: divide problem into sub-problems
  - with a manageable number of sub-problems
- Find maximal partial configurations up to $T_1$, then $T_2$, $T_3$, and $T_4$

**Viterbi Algorithm Example**

The idea of the Viterbi algorithm is to incrementally calculate maximal values of the following parts of the above product:

$$\mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1)$$

for all possible values of $T_1$, then

$$\mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1) \cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = *|T_2)$$

for all possible values of $T_2$ and so on. The computation can be summarized in the following table:

| | $T_1$ ($W_1$ = flies) | $T_2$ ($W_2$ = *) | $T_3$ ($W_3$ = like) | $T_4$ ($W_4$ = flies) |
|---|---|---|---|---|
| | $\mathrm{P}(T_1)\mathrm{P}(W_1\|T_1)$ | $p \cdot \mathrm{P}(T_2\|T_1)\mathrm{P}(W_2\|T_2)$ | $p \cdot \mathrm{P}(T_3\|T_2)\mathrm{P}(W_3\|T_3)$ | $p \cdot \mathrm{P}(T_4\|T_3)\mathrm{P}(W_4\|T_4)$ |
| D | $0 \times 0 = 0$ | DD: $0 \times 0 \times \frac{1}{3} = 0$<br>ND: $\frac{1}{9} \times 0 \times \frac{1}{3} = 0$<br>PD: 0<br>VD: 0<br>max: 0 | DD: $0 \times 0 \times 0 = 0$<br>ND: $\frac{1}{90} \times 0 \times = 0$<br>PD: $\frac{1}{50} \times \frac{1}{2} \times 0 = 0$<br>VD: $\frac{1}{90} \times 0 \times 0 = 0$<br>max: 0 | DD: $0 \times 0 \times 0 = 0$<br>ND: $0 \times 0 \times 0 = 0$<br>PD: $\frac{1}{225} \times 0.5 \times 0 = 0$<br>VD: $0 \times 0 \times 0 = 0$<br>max: 0 |
| N | $0.5 \times \frac{2}{9} = \frac{1}{9}$ | DN: $0 \times 1 \ldots = 0$<br>NN: $\frac{1}{9} \times 0 \ldots = 0$<br>PN: $0 \times \ldots = 0$<br>VN: $0.2 \times 0.5 \times \frac{1}{9} = \frac{1}{90}$<br>max: $\frac{1}{90}$ | DN: $0 \times 1 \times 0 = 0$<br>NN: $\frac{1}{90} \times 0 \ldots = 0$<br>PN: $\frac{1}{50} \times 0.5 \times 0 = 0$<br>VN: $\frac{1}{90} \times 0.5 \times 0 = 0$<br>max: 0 | DN: $0 \times 1 \times \frac{2}{9} = 0$<br>NN: $0 \times 0 \times \frac{2}{9} = 0$<br>PN: $\frac{1}{225} \times 0.5 \times \frac{2}{9} = \frac{1}{2025}$<br>VN: $0 \times 0.5 \times \frac{2}{9} = 0$<br>max: $\frac{1}{2025}$ |
| P | $0 \times 0 = 0$ | DP: $0 \times \ldots = 0$<br>NP: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$<br>PP: $0 \times \ldots = 0$<br>VP: $0.2 \times 0.5 \times 0.2 = \frac{1}{50}$<br>max: $\frac{1}{50}$ | DP: $0 \times 0 \times 0.8 = 0$<br>NP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$<br>PP: $\frac{1}{50} \times 0 \times 0.8 = 0$<br>VP: $\frac{1}{90} \times 0.5 \times 0.8 = \frac{1}{225}$<br>max: $\frac{1}{225}$ | DP: $0 \times 0 \times 0 = 0$<br>NP: $0 \times 0.5 \times 0 = 0$<br>PP: $\frac{1}{225} \times 0 \times 0 = 0$<br>VP: $0 \times 0.5 \times 0 = 0$<br>max: 0 |
| V | $0.5 \times 0.4 = 0.2$ | DV: $0 \times \ldots = 0$<br>NV: $\frac{1}{9} \times 0.5 \times 0.2 = \frac{1}{90}$<br>PV: $0 \times \ldots = 0$<br>VV: $0.2 \times 0 \ldots = 0$<br>max: $\frac{1}{90}$ | DV: $0 \times 0 \times 0 = 0$<br>NV: $\frac{1}{90} \times 0.5 \times 0 = 0$<br>PV: $\frac{1}{50} \times 0 \times 0 = 0$<br>VV: $\frac{1}{90} \times 0 \times 0 = 0$<br>max: 0 | DV: $0 \times 0 \times 0.4 = 0$<br>NV: $0 \times 0.5 \times 0.4 = 0$<br>PV: $\frac{1}{225} \times 0 \times 0.4 = 0$<br>VV: $0 \times 0 \times 0.4 = 0$<br>max: 0 |

The table is filled column by column. We can see now that the largest value that the expression

$$\mathrm{P}(T_1) \cdot \mathrm{P}(W_1 = \text{flies}|T_1) \cdot \cdot \mathrm{P}(T_2|T_1) \cdot \mathrm{P}(W_2 = *|T_2) \cdot \mathrm{P}(T_3|T_2) \cdot \mathrm{P}(W_3 = \text{like}|T_3) \cdot \mathrm{P}(T_4|T_3) \cdot \mathrm{P}(W_4 = \text{flies}|T_4)$$

can obtain is $\frac{1}{2025}$, and is achieved with $T_4$ = N. If we work backwards through the table, we can obtain the optimal values for previous variables as well: $T_3$ = P, $T_2$ = V, and $T_1$ = N. We can also choose $T_2$ = N, but in this case we have $T_1$ = V.