Natural Language Processing CSCI 4152/6509 — Lecture 11 N-gram Model and Markov Chain Model

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Location: Studley LSC-Psychology P5260

Previous Lectures

- Joint Distribution and Fully Independent Model review
- Classification example:
  - Joint Distribution Model
  - Fully Independent Model
  - Naïve Bayes Model
- Naïve Bayes classification model
  - Assumption, definition, graphical representation
  - Number of parameters
  - Pros and cons, additional notes
  - Bernoulli and Multinomial Naïve Bayes

N-gram Model

- What is *Language Modeling*
- *Language Modeling:* Estimating probability of arbitrary NL sentence: P(sentence)
- Alternative definition: Predicting the most likely next word
- N-gram model is a fundamental and intuitive model for this task
- Large Language Models more recently were directly influenced by this previous definition

Speech Recognition Motivation

- Original motivation for Language Modeling comes from Speech Recognition

$$\begin{aligned}
\arg\max_{\text{sentence}} \mathrm{P}(\text{sentence}|\text{sound}) &= \arg\max_{\text{sentence}} \frac{\mathrm{P}(\text{sentence}, \text{sound})}{\mathrm{P}(\text{sound})} \\
&= \arg\max_{\text{sentence}} \mathrm{P}(\text{sentence}, \text{sound}) \\
&= \arg\max_{\text{sentence}} \mathrm{P}(\text{sound}|\text{sentence})\mathrm{P}(\text{sentence})
\end{aligned}$$

- Acoustic model and Language model

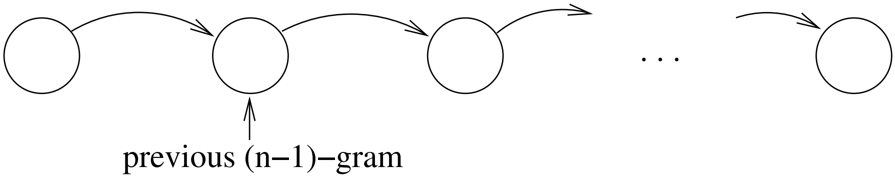
N-gram Language Model

- Predict next word using $(n-1)$ previous words
- Example assumption with $n = 3$:

$$
\begin{aligned}
\mathrm{P}(w_1 w_2 \ldots w_k) = \\
& \mathrm{P}(w_1 | \cdot\,\cdot) \mathrm{P}(w_2 | w_1 \cdot) \mathrm{P}(w_3 | w_2 w_1) \ldots \\
& \mathrm{P}(w_k | w_{k-1} w_{k-2})
\end{aligned}
$$

N-gram Model: Notes

- Reading: Chapter 4 of [JM]
- Use of log probabilities
  - similarly as in the Naïve Bayes model for text
- Graphical representation



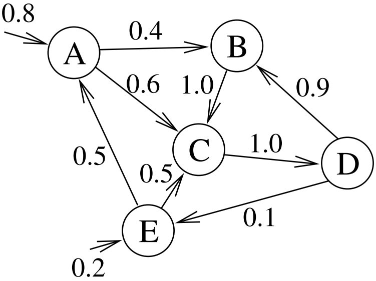
N-gram Model as a Markov Chain

- N-gram Model is very similar to Markov Chain Model
- Markov Chain consists of
  - sequence of variables V_1, V_2, ...
  - probability of V_1 is independent
  - each next variable is dependent only on the previous variable: V_2 on V_1, V_3 on V_2, etc.
  - Conditional Probability Tables: $P(V_1)$, $P(V_2|V_1)$, ...
- Markov Chain is identical to bi-gram model, but higher-order n-gram models are very similar as well

Markov Chain: Formal Definition

- *Stochastic process* is a family of variables $\{V_i\}\ i \in I$, $\{V_i, i \in I\}$, or $\{V_t, t \in T\}$
- *Markov process:* for any t, and given V_t, the values of V_s, where $s > t$, do not depend on values of V_u, where $u < t$.
- If I is finite or countably infinite: V_i depends only on V_{i-1}
- In this case Markov process is called *Markov chain*
- Markov chain over a finite domain can be represented using a DFA (Deterministic Finite Automaton)

Markov Chain: Example



This model could generate the sequence $\{A, C, D, B, C\}$ of length 5 with probability:

$$0.8 \cdot 0.6 \cdot 1.0 \cdot 0.9 \cdot 1.0 = 0.432$$

assuming that we are modelling sequences of this length.

Evaluating Language Models: Perplexity

- Evaluation of language model: extrinsic and intrinsic
- Extrinsic: model embedded in application
- Intrinsic: direct evaluation using a measure
- Perplexity, W — text, $L = |W|$,

$$\mathrm{PP}(W) = \sqrt[L]{\frac{1}{P(W)}} = \sqrt[L]{\prod_i \frac{1}{P(w_i|w_{i-n+1}\ldots w_{i-1})}}$$

- Weighted average branching factor

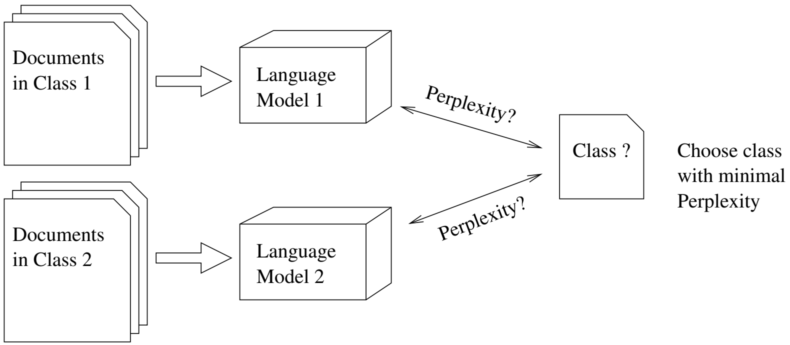
Use of Language Modeling in Classification

- Perplexity, W — text, $L = |W|$,

$$\text{PP}(W) = \sqrt[L]{\frac{1}{P(W)}} = \sqrt[L]{\prod_i \frac{1}{P(w_i|w_{i-n+1}\ldots w_{i-1})}}$$

- Text classification using language models

Classification using Language Modeling



Unigram Model and Multinomial Naïve Bayes

- It is interesting that classification using Unigram Language Model is same as Multinomial Naïve Bayes with all words

N-gram Model Smoothing

- Smoothing is used to avoid probability 0 due to sparse data
- Some smoothing methods:
  - Add-one smoothing (Laplace smoothing)
  - Witten-Bell smoothing
  - Good-Turing smoothing
  - Kneser-Ney smoothing (new edition of [JM])

Example: Character Unigram Probabilities

- Training example: `mississippi`
- What are letter unigram probabilities?
- What would be probability of the word '`river`' based on this model?

Unigram Probabilities: `mississippi`

Add-one Smoothing (Laplace Smoothing)

- Idea: Start with count 1 for all events
- $|V|$ = vocabulary size (unique tokens)
- n = length of text in tokens
- Smoothed unigram probabilities:

$$P(w) = \frac{\#(w) + 1}{n + |V|}$$

- Smoothed bi-gram probabilities

$$P(a|b) = \frac{\#(ba) + 1}{\#(b) + |V|}$$

Mississippi Example: Add-one Smoothing

- Let us again consider the example trained on the word: `mississippi`
- What are letter unigram probabilities with add-one smoothing?
- What is the probability of: `river`

Mississippi Example: Add-one Smoothing