Faculty of Computer Science, Dalhousie University *21-Oct-2025*

CSCI 4152/6509 — Natural Language Processing

Lecture 8: Similarity Based Classification

Location: Studley LSC-Psychology P5260 Instructor: Vlado Keselj
Time: 14:35 – 15:55

Previous Lecture

- IR evaluation measures review
- Recall-precision curve review
- Text classification review
- Evaluation measures for Text Classification review

9.4 Evaluating Classifiers

In order to decide which machine learning classification method is the best choice for a classification task, we need to evaluate those methods, or classifiers for short. The goal of evaluation is to see what kind of performance we can expect from a classifier after it learns its model from training data. The two main concerns with classifier learning are underfitting and overfitting.

9.4.1 Underfitting and Overfitting

Slide notes:

Evaluation Methods for Classification
- General issues in classification
  - Underfitting and Overfitting
- Example with polynomial-based function learning
  - Underfitting and Overfitting

Two basic issues that we can find with classifiers that are trained from labeled data are *underfitting* and *overfitting*. *Underfitting* is an issue in which the training algorithm does not learn well from training data, which is indicated by poor performance on the training data itself. This means that it cannot *fit* well the training data. In this case, we cannot expect any better performance on unseen data. *Overfitting* is the issue that the classifiers fits very well the training data, and even too well in sense that it learns signals from training data that are not relevant to the task, which hurt its performance on unseen data. In other words, the classifier fits the training data too well, and thus it does not properly generalize in order to classify unseen data. These issues can be well illustrated on an example of polynomial-based function learning.

Polynomial-based Function Learning Example. An example for overfitting and underfitting can be created from function learning. We can draw a set of, for example, 10 points in a 2-dimensional plane, being closely distributed around a parabola with equation $y = x^2/4$. Assuming that we do not know that the points are generally close to this parabola, we could try to fit them to polynomials of various degrees. By fitting them with a polynomial with the first degree, i.e., a line, we can see that the data does not fit the line very well. We call this a training error, since our line is learned from the 10 training points, and this is an example of underfitting, since we cannot fit the training data. On the other hand, if we try a polynomial of 9th degree, we can perfectly fit the training data, but the polynomial

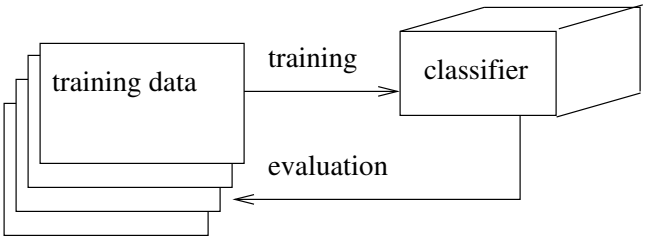
will not match well new data. This is an overfitting error, since the learned function fits the training data too well, fitting even the errors which may have happened in measurements or similar.

9.4.2 Evaluation Methods for Text Classifiers

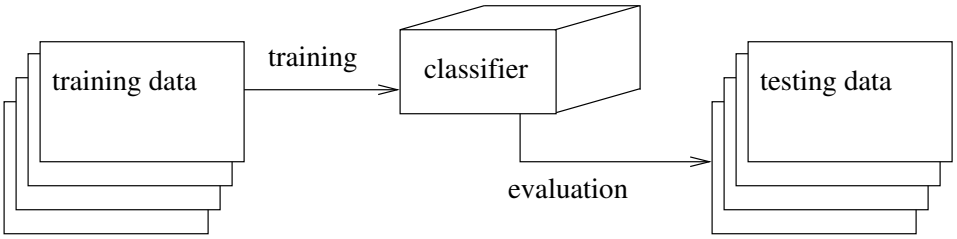
Slide notes:

Evaluation Methods for Text Classifiers
- Training Error
- Train and Test
- N-fold Cross-validation

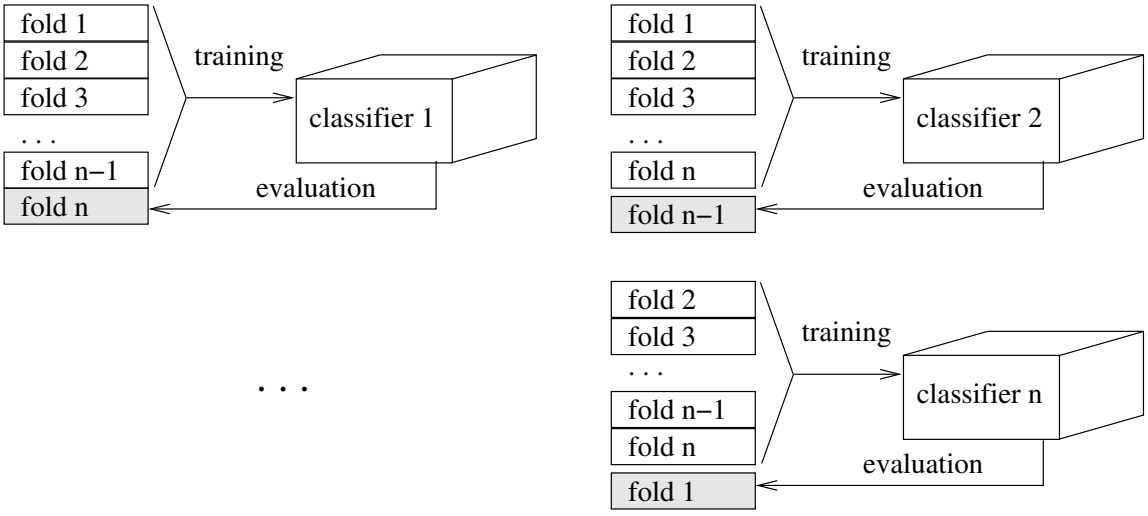
Training Error. The classifier is trained on a training data set and also evaluated on the same data set. It is a good idea to get this result, although it is obviously *biased* towards the training data. This evaluation can detect underfitting but not overfitting of the training data.



Train and test. The data is divided into two parts: training and testing part. The split is usually 90% for training and 10% for testing, but sometimes 2/3 of data is used for training and 1/3 for testing. This is an unbiased evaluation, which can detect underfitting as well as overfitting. To be sure that the evaluation is unbiased, it is important not to use testing data in any way, even to glance at it, if it may influence our decisions regarding classifier construction. With some methodologically generic methods, this is not an issue.



N-fold cross-validation. In this method, the data is randomly partitioned into n equal parts. n experiments are performed, where in each another part is taken as the testing data, while remaining parts are used for training. At the end, the results over experiments are averaged. This is unbiased testing that gives more statistical significance than train-and-test, but it is not applicable if we need to examine the training data during classifier construction.



Side note: An interesting link:

– SpamAssassin is considered the best, or at least one of the best spam-classification software packages. It is an open-source package available at:
`http://spamassassin.apache.org`

9.5 Text Clustering

Note: *This sub-subsection (Text Clustering) is not covered in this course. These notes are only for your additional information, or use in a project.*

- Task definition
- Example of unsupervised learning
- Example approach: the simple k-means approach
- Hierarchical clustering
  - agglomerative, and
  - divisive
- Evaluation
  - inter-cluster similarity (average inter-cluster distance)
  - cluster purity (classes known)
  - entropy or information gain (classes known)

Simple k-means. In the simple k-means clustering, all documents are translated into vectors of same dimensionality. We choose a number of clusters k in advance and choose randomly initial k centroids, i.e., vectors of the same dimensionality as the documents. After this we repeat the following two steps:

1) Assign each document to the closest centroid using Euclidean distance. In this way, all documents are partitioned into k clusters.

2) Calculate new k centroids as centroids of k clusters created in the previous step. A centroid of a set of vectors is equivalent to the arithmetic mean of a set of numbers, i.e., for n vectors $v_1, v_2, \ldots, v_n$, their centroid is calculated using the formula:

$$\textit{centroid} = \frac{\sum_{i=1}^{n} v_i}{n}$$

This iterative process stops either if: (1) the clusters do not change between two iterations, i.e., a stable partition is achieved; (2) a clustering quality measure stops improving; or, (3) we reach a predefined iteration limit.

Clustering evaluation. The clustering evaluation methods can be *internal* and *external.* In the internal methods, clusters are evaluated based on the high similarity among items in the same cluster, and low similarity among items in different clusters. In the external methods, clustering is evaluated as part of a larger application, and the performance of that application is measured.

Average inter-cluster distance. As an example for internal evaluation, we could find the average distance among all pairs of items belonging to the same cluster. This is the average inter-cluster distance. The lower the average inter-cluster distance, the better is the clustering. The number of clusters needs to be limited from above in this case, since one-item-one-cluster approach would lead to the best (zero) average inter-cluster distance, with a very large number of clusters.

Clustering purity. The clustering purity measure can be used if we have document labels according to some classification. We assume that a better clustering will group more documents with the same label in one cluster, and tend to put documents with different labels in different clusters. Clustering purity evaluation can be described as follows: we assign each cluster its majority class and then we measure accuracy as it would be done for classification:

$$\text{purity}(\Omega, C) = \frac{1}{N}\Sigma_k \max_j |\omega_k \cup c_j|$$

where $\Omega = \{\omega_1, \ldots, \omega_K\}$ is the set of clusters, and $C = \{c_1, \ldots c_J\}$ is the set of classes.

There are other evaluation measures for clustering, including F-measure for clustering.

10 Similarity-based Text Classification

In this section we will discuss a simple approach to text classification using similarity measures between test document and training documents in different classes. We call these general approach the similarity-based classification.

Profile Similarity Text Classification

- Aggregate training text for each class into a profile
- Aggregate testing text into another profile
- Classify according to profile similarity
- If a profile is a vector, we can use different similarity measures; e.g.,
  - cosine similarity,
  - Euclidean similarity, or
  - some other type of vector similarity

10.1 Similarity-based Classification using Vector Space Model

There are different approaches to creating a vector from a document or a set of documents. We talked about the Vector Space Model, according to which we can create boolean vectors, term frequency vectors, or *tfidf* vectors. One can use the 'word2vec' model[1] or another similar word-embedding model for translating words to vectors, which can then be used to generate document vectors.

When it comes to the similarity measure between vectors, one common measure is the *cosine similarity,* based on the cosine angle calculation between vectors in vector spaces. For two vectors $\mathbf{a}_1 = (a_1, a_2, \ldots, a_n)$ and

[1] https://en.wikipedia.org/wiki/Word2vec

$\mathbf{b}_1 = (b_1, b_2, \ldots, b_n)$ the cosine similarity between them is calculated using the formula:

$$cosine\_sim(\mathbf{a}, \mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||} = \frac{\sum_i^n a_i b_i}{\sqrt{\sum_i^n a_i^2} \cdot \sqrt{\sum_i^n b_i^2}} = \frac{a_1 b_1 + a_2 b_2 + \ldots a_n b_n}{\sqrt{a_1^2 + a_2^2 + \ldots + a_n^2} \cdot \sqrt{b_1^2 + b_2^2 + \ldots + b_n^2}}$$

The cosine similarity is always between 0 and 1, with numbers close to 0 meaning that the vectors are very different, and numbers close to 1 meaning that the vectors are very similar.

The *Euclidean distance* is another very common similarity measure. It comes form the point distance in the Euclidean space; i.e., our usual physical space. It is usually applied to normalized vectors; i.e., vectors scaled to length 1. The Euclidean distance formula is:

$$Euclidean\_sim(\mathbf{a}, \mathbf{b}) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2} = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots (a_n - b_n)^2}$$

10.2 Common N-Grams Method for Text Classification (CNG)

We will now take a closer look at a specific kind of text classification problem, called *authorship attribution,* and a simple character n-grams based method that works well on this task, called the *CNG method* (Common N-Gram analysis method). The method was initially published in 2003 and used in the authorship attribution task, but later showed some good results on other tasks as well. Figure 2 illustrates the general authorship attribution problem.

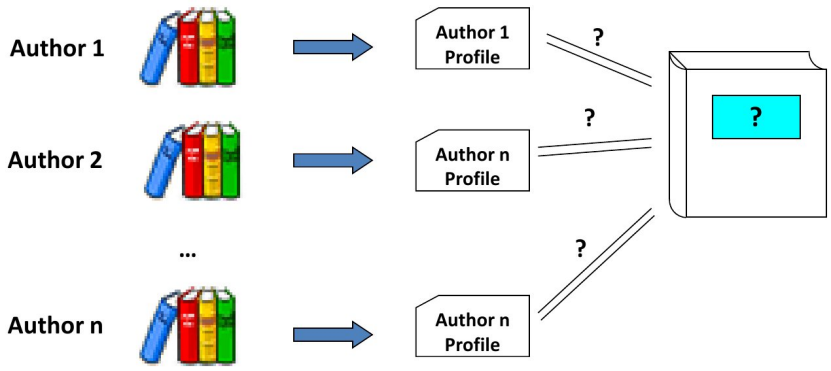


Figure 2: Authorship Attribution Problem

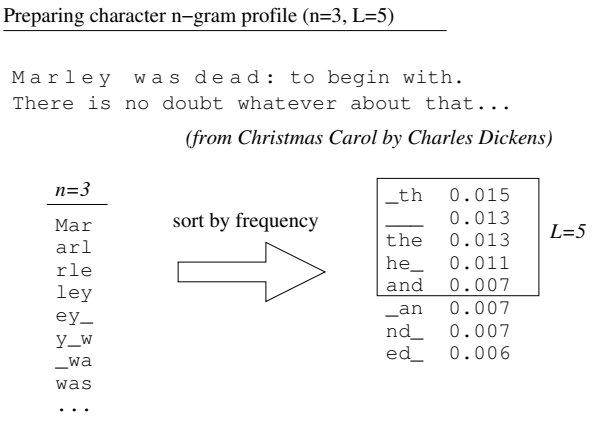
CNG Method Overview

- Method based on character n-grams
- Language independent
- Based on creating n-gram based author profiles
- Similarity based (a type of kNN method—k Nearest Neighbours)
- Similarity measure:

$$\sum_{g \in D_1 \cup D_2} \left(\frac{f_1(g) - f_2(g)}{\frac{f_1(g) + f_2(g)}{2}} \right)^2 = \sum_{g \in D_1 \cup D_2} \left(\frac{2 \cdot (f_1(g) - f_2(g))}{f_1(g) + f_2(g)} \right)^2 \quad (1)$$

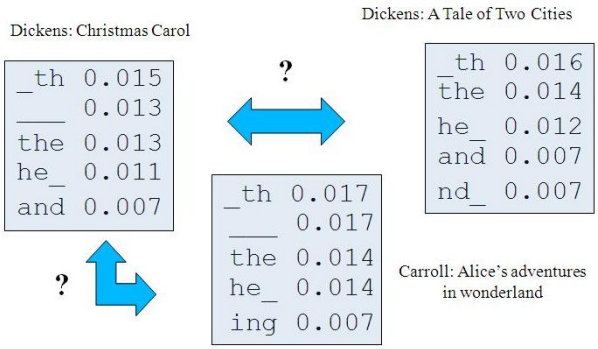
where $f_i(g) = 0$ if $g \notin D_i$.

Example of Creating an Author Profile



The profile is created by collecting all character n-grams of certain size, sorting them according to the normalized frequency; i.e., frequency obtained by taking n-grams count divided by the total number of n-grams; and keeping L most frequent n-grams, where L is a positive integer called the profile length.

How to measure profile similarity?



CNG Similarity Measure

- Euclidean-style distance with relative differences, rather than absolute
- Example: instead of using $0.88 - 0.80 = 0.10$, we say it is about 10% difference, which is the same for 0.088 and 0.080
- To be symmetric, divide by the arithmetic average:

$$d(f_1, f_2) = \Sigma_{n \in \text{dom}(f_1) \cup \text{dom}(f_2)} \left(\frac{f_1(n) - f_2(n)}{\frac{f_1(n)+f_2(n)}{2}} \right)^2$$

- $dom(f_i)$ is the domain of function f_i, i.e., of the profile i

Motivation for Similarity Measure: The idea for this particular similarity measure comes from the standard Euclidean distance:

$$\sum_{g \in D_1 \cup D_2} (f_1(g) - f_2(g))^2 \tag{2}$$

However, the Euclidean distance would be dominated by the most frequent n-grams, since their frequency is orders of magnitude higher than lower frequency n-grams. This is due to a Zipf's-like distribution law for n-grams; i.e., a power-law distribution of n-gram frequencies. To increase the impact of lower-frequency n-grams, we calculate

an Euclidean-style distance for relative n-gram frequency differences. This is how we obtain the CNG similarity measure:

$$\sum_{g \in D_1 \cup D_2} \left(\frac{f_1(g) - f_2(g)}{\frac{f_1(g)+f_2(g)}{2}} \right)^2 = \sum_{g \in D_1 \cup D_2} \left(\frac{2 \cdot (f_1(g) - f_2(g))}{f_1(g) + f_2(g)} \right)^2$$

where $f_i(g) = 0$ if $g \notin D_i$. It is important that we take a union of the n-grams in profiles, rather than an intersection, since taking an intersection would lead to a low distance value for profiles with small overlap, which are intuitively dissimilar.

CNG Similarity Example

Let us consider an example of comparing two very simple documents, each one consisting of one line:

```
d1: the dog eat homework
d2: the cat eat homework
```

In other words, the first document `d1` contains only the string '`the dog eat homework`" and the second document `d2` contains the string "`the cat eat homework`". The two strings are not grammatical sentences for simplicity reasons, although they may realistically occur after stemming grammatical sentences. If we collect all character tri-grams from these strings, we will obtain the following trigrams from the first document:

```
the  he_  e_d  _do  dog  og_  g_e  _ea  eat
at_  t_h  _ho  hom  ome  mew  ewo  wor  ork
```

If we sort the n-grams, count them, and normalize their frequency we obtain the following results:

Trigram	count	normalized frequency (f_1)
_do	1	0.0555555555555556
_ea	1	0.0555555555555556
_ho	1	0.0555555555555556
at_	1	0.0555555555555556
dog	1	0.0555555555555556
e_d	1	0.0555555555555556
eat	1	0.0555555555555556
ewo	1	0.0555555555555556
g_e	1	0.0555555555555556
he_	1	0.0555555555555556
hom	1	0.0555555555555556
mew	1	0.0555555555555556
og_	1	0.0555555555555556
ome	1	0.0555555555555556
ork	1	0.0555555555555556
t_h	1	0.0555555555555556
the	1	0.0555555555555556
wor	1	0.0555555555555556
sum	18	1.0

The list of frequencies is very simply since we have no repeated trigrams in this simple string. Similarly, for the second string we obtain frequencies:

Trigram	count	normalized frequency (f_2)
_ca	1	0.0555555555555556
_ea	1	0.0555555555555556
_ho	1	0.0555555555555556
at_	2	0.111111111111111
cat	1	0.0555555555555556
e_c	1	0.0555555555555556
eat	1	0.0555555555555556
ewo	1	0.0555555555555556
he_	1	0.0555555555555556
hom	1	0.0555555555555556
mew	1	0.0555555555555556
ome	1	0.0555555555555556
ork	1	0.0555555555555556
t_e	1	0.0555555555555556
t_h	1	0.0555555555555556
the	1	0.0555555555555556
wor	1	0.0555555555555556
sum	18	1.0

Since the documents are very short, we are not going to use the profile cut-off length L; i.e., we will use all n-grams. In order to calculate the CNG distance, we now make a union of all n-grams and compare their frequencies. This is how we obtain the following table:

Trigram	f_1	f_2	$(2(f_1 - f_2)/(f_1 + f_2))^2$
_ca	0	0.0555555555555556	4.0
_do	0.0555555555555556	0	4.0
_ea	0.0555555555555556	0.0555555555555556	0.0
_ho	0.0555555555555556	0.0555555555555556	0.0
at_	0.0555555555555556	0.111111111111111	0.444444444444444
cat	0	0.0555555555555556	4.0
dog	0.0555555555555556	0	4.0
e_c	0	0.0555555555555556	4.0
e_d	0.0555555555555556	0	4.0
eat	0.0555555555555556	0.0555555555555556	0.0
ewo	0.0555555555555556	0.0555555555555556	0.0
g_e	0.0555555555555556	0	4.0
he_	0.0555555555555556	0.0555555555555556	0.0
hom	0.0555555555555556	0.0555555555555556	0.0
mew	0.0555555555555556	0.0555555555555556	0.0
og_	0.0555555555555556	0	4.0
ome	0.0555555555555556	0.0555555555555556	0.0
ork	0.0555555555555556	0.0555555555555556	0.0
t_e	0	0.0555555555555556	4.0
t_h	0.0555555555555556	0.0555555555555556	0.0
the	0.0555555555555556	0.0555555555555556	0.0
wor	0.0555555555555556	0.0555555555555556	0.0
sum			36.444444444444444

Hence, the CNG distance between documents d1 and d2 is 36.444444444444444.

Classification using CNG

- Create profile for each class using training text
  - done by merging all texts in each class into one long document
  - another option: centroid of profiles of individual documents
- Create profile for the test document
- Assign class to the document according to the closest class profile according to the CNG distance

10.3 Edit Distance

The CNG similarity is one way of measuring text similarity, which is quite robust to typos, morphological variations, and similar general string differences. It also somewhat captures word ordering and punctuation, since n-grams can span two words. These characteristics are particularly noticable when comparing this similarity to the standard bag-of-words approach, which may or may not use stemming, and which relies on cosine similarity. Another similarity measure that is very string-oriented, with a similar set of characteristics, is the *edit distance.*

Slide notes:

Edit Distance: Introduction

- Edit distance is a similarity measure convenient for words and short texts, robust for typos and morphological differences
- Tends to be too expensive for longer texts
- Consider typical errors that cause typos:
  - there $\rightarrow$ thre (missed a letter)
  - there $\rightarrow$ theare (inserted an extra letter)
  - there $\rightarrow$ yhere (mistyped a letter)
- Task: find a word in lexicon most likely to produce incorrect word found in text

Slide notes:

Edit Distance: Brute Force Approaches

- one approach: search lexicon and try deleting, inserting, and replacing each of the letters, and compare with mistyped word
- this is already quite expensive, but what with multiple errors?
- Can we find the minimal number of edit operations (deletes, inserts, or substitutions) that would lead from a source string s to the target string t?
- This is *minimal edit distance* — it always exists because we can always delete $|s|$ letters and insert $|t|$ letters, so it is always $\leq |s| + |t|$

Slide notes:

Edit Distance: Properties

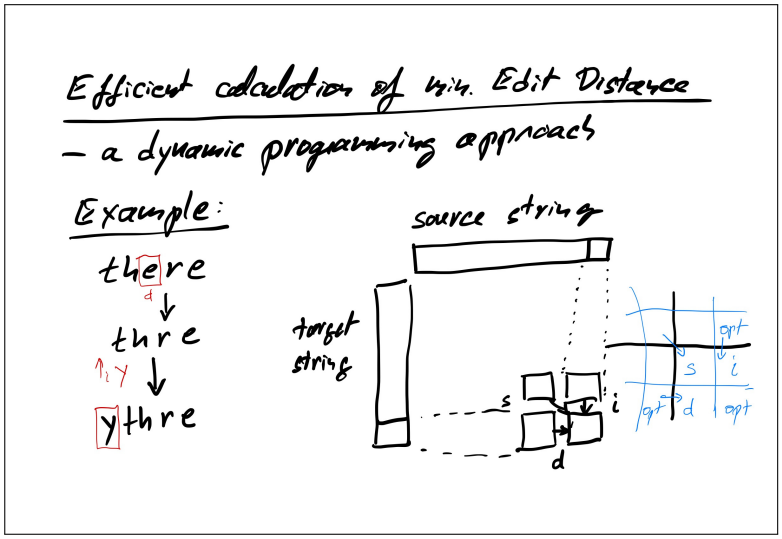
- Reflexive: $d(s,t) = 0$ if and only if $s = t$
- Symmetric: $d(s,t) = d(t,s)$, because edit operations are reversible
- Transitive: $d(s,t) + d(t,v) \geq d(s,v)$
- Can be parametrized with $cost\_d(c)$, $cost\_i(c)$, $cost\_s(c,d)$ for all characters c and d; positive cost functions with exception $cost\_s(c,c) = 0$
- If cost is 1 for delete and insert, and 2 for substitute operations, it is also known as the Levenshtein distance [JM] (all cost$= 1$ according to some sources)

Slide notes:

Edit Distance: Dynamic Programming Idea

- calculate optimal distance between $s = xe$ and $t = yf$ using optimal distances between xe and y, x and yf, and x and y

Slide notes:



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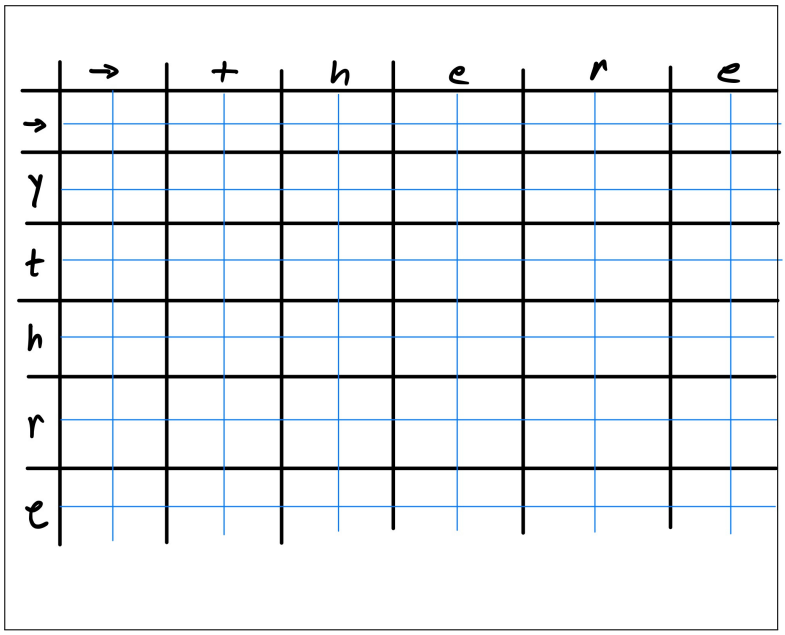
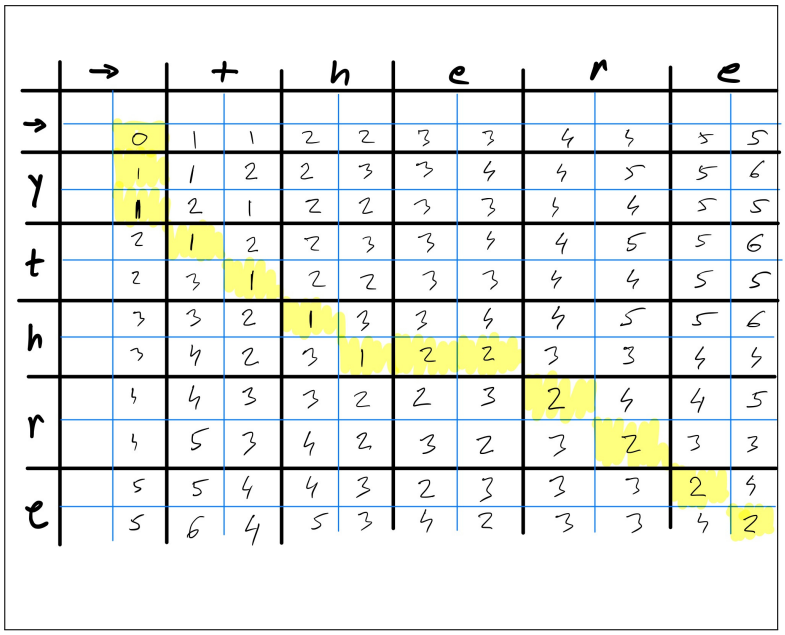


Slide notes:



Edit Distance Algorithm

```
Algorithm EditDistance(s,t)
n = len(s); m  = len(t)
d[m+1,n+1] - initialize to 0s
for i=1 to n do d[0,i] = d[0,i-1] + cost_d(s[i-1])
for j=1 to m do d[j,0] = d[j-1,0] + cost_i(t[j-1])
for j=1 to m do
  for i=1 to n do
    d[j,i] = min( d[j-1,i-1] + cost_s(s[i-1],t[j-1]),
                  d[j-1,i]   + cost_i(t[j-1]),
                  d[j,i-1]   + cost_d(s[i-1]) )
return d[m,n]
```